Practical Rheology Section 3

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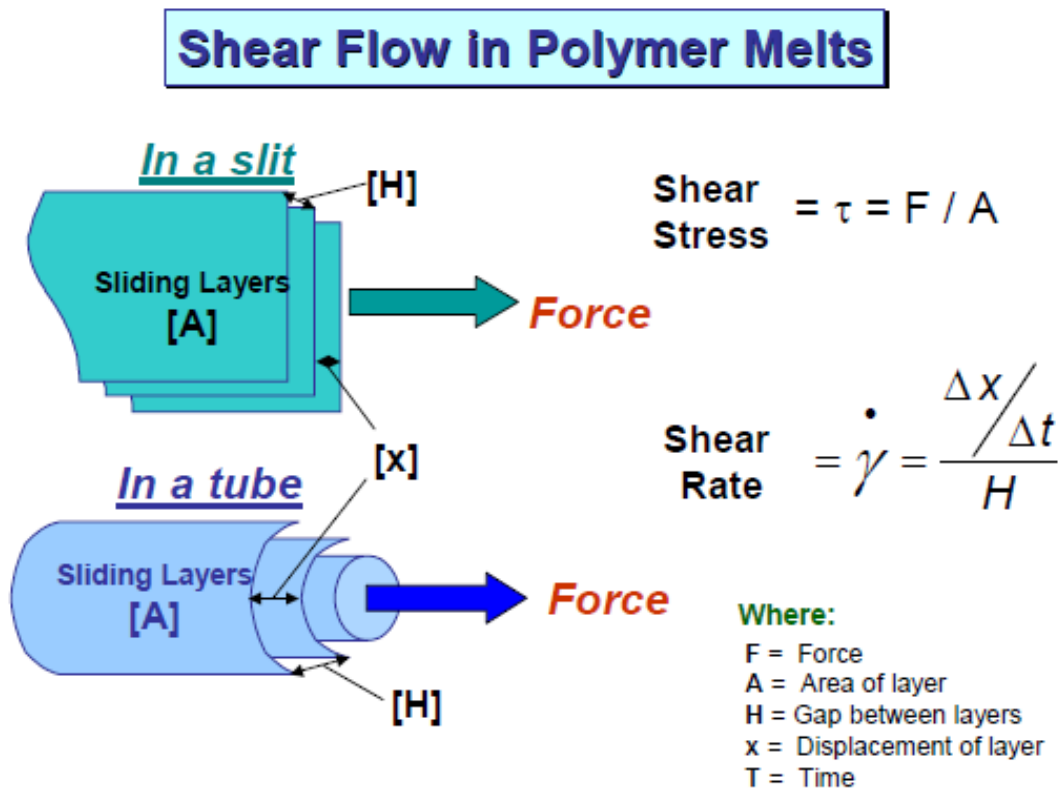**www.dynisco.com**

CALCULATING FLOW DATA

Some important assumptions are made in the analysis of flow in a capillary rheometer. Of these the most important are:
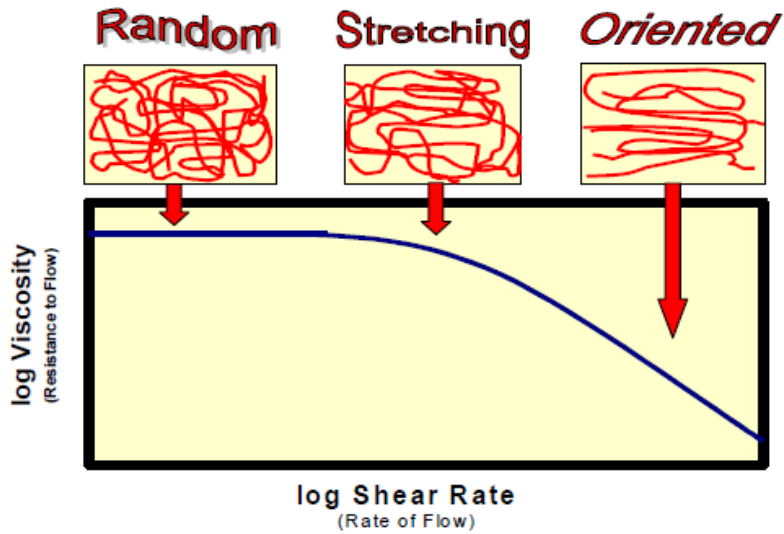
1. The flow is isothermal.
2. There is no slip at the capillary wall.
3. The melt is incompressible.
4. The flow pattern is constant through the die. ("Fully developed flow".)
5. Dissipation of energy at the die entrance, or due to chain uncoiling, is negligible.
(See the next chapter.)

13.1 Basic Equations.

The most common type of flow occurring in the processing of polymer melts is shear flow. In this type of flow, one may visualize that one layer of melt flows over another on application of a shearing force. The process is described by the relationship between two variables, the shear rate and the shear stress. The shear rate is usually designated by

Shear Flow in Polymer Melts

In a slit
[H]
Sliding Layers [A]
→ Force
[x]

In a tube
Sliding Layers [A]
→ Force
[H]

Shear Stress $= \tau = F / A$

Shear Rate $= \dot{\gamma} = \dfrac{\Delta x / \Delta t}{H}$

Where:
F = Force
A = Area of layer
H = Gap between layers
x = Displacement of layer
T = Time

Shear Flow in Polymer Melts

the Greek letter gamma with a dot above the letter, that is, gamma dot or . Shear rate is given in reciprocal seconds (that is, in 1/sec or sec $\bullet\gamma$ -1). The larger the number quoted the faster the material is being sheared or forced to flow. The shear stress, which is usually designated by the Greek letter τ, is the stress (i.e. the force per unit area) required to cause one layer to flow over another at the required rate. If the assumptions made above are considered to be valid then it may be shown that:

τ = The Shear Stress at the Wall of the Die = PRc/2Lc,

$\gamma\bullet$ a,w = The Shear Rate = [4Q (3n*+1)] / [πR3
c (4n*)]

Where:

n* = Δlog(RcP/2Lc) / Δlog(4Q/BR3
c)

$\bullet\gamma$

w = the shear rate at the wall of the die.

τ = the shear stress. Q = the volumetric flow rate.

P = the measured pressure.

Rc = the die radius.

Rb = the barrel radius

Lc = the die length.

Flow Curves.

The expression for the shear rate at the wall of the die is rather cumbersome, so in practice it is just as useful to use the expression

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γ• a,w = 4Q / πR3

c where, •γ a,w is known as the apparent shear rate at the wall. It may be shown that, as with the true shear rate, there is a unique relationship (i.e. just one relationship) between shear stress and •γ a,w and this simplified expression may be used for scale up work. Hence the "flow curves" provided by most laboratories are generally obtained by plotting τ or η (viscosity) against •γ a,w. The wall of the tube is, however, simply a place at which the shear stress and shear rate may be calculated from knowledge of flow rates and pressures. The relationship between τ and •γ a,w will, however, be independent of the position in the flow system and equally applicable to flow in other geometries such as slits. For this reason it is quite correct to plot τ against •γ a,w without printing the subscripts. Occasionally shear rate is denoted by γ without the dot

Viscosity Curves for Some Commodity Thermoplastics
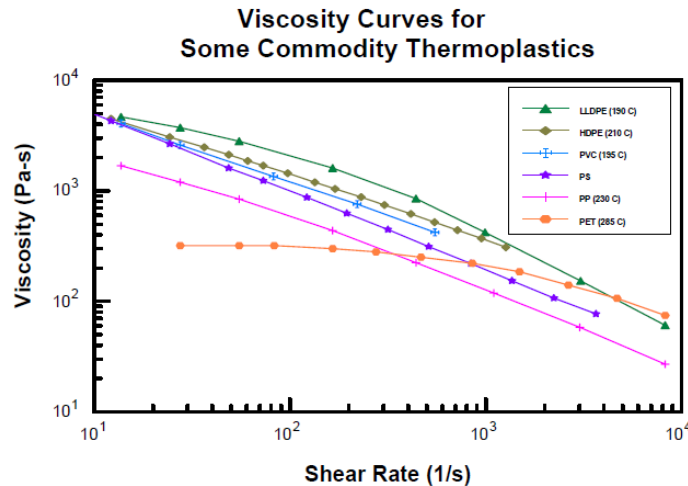
above it. This should not be encouraged since this symbol is widely used to indicate elastic shear strain. By plotting shear stress (Pa) against shear rate (1/sec) flow curves may be obtained. Viscosity (Pa-sec) is obtained by dividing shear stress by shear rate. Measuring γ• A,W and τ Let the ram speed = S (measured in mm/s) and the barrel diameter = D (measured in mm). Then the volumetric flow rate

Q = S (πD2

b/4) (Equation1).

This is the same in both the barrel and the capillary. Now the apparent shear rate at wall is γ• a,w = 4Q/πR³c (Equation 2). Where Rc is the radius of the capillary in mm.

Substituting equation 1 into equation 2 and simplifying we obtain:

γ• a,w = S D2

b / R³c = 4 S (R2

b / R3

c)

The barrel diameter for the Kayeness capillary rheometer is 0.375", i.e. 9.550 mm.

In this case the formula becomes:

γ•a,w = 91.203 S / R³c

when the capillary radius is in mm and the ram speed is in mm/s.

Since the shear stress at the wall is given by
τ = P Rc / 2Lc (Equation 4).
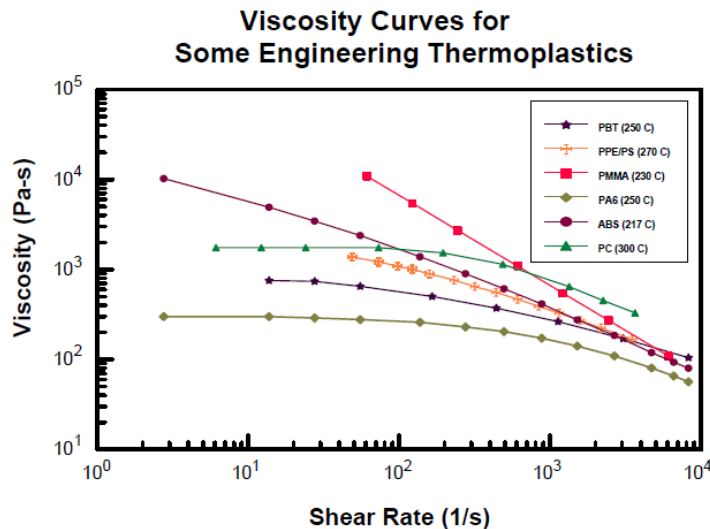Then, for a 20:1 L/D die, τ = P/80 (where P is the pressure at the entrance to the capillary.)

Apparent Viscosity.

By plotting shear stress (Pa) against shear rate (1/sec) flow curves may be obtained. Viscosity (Pa-sec) is obtained by dividing shear stress by shear rate. Dividing τ (the shear stress) by $•\gamma$ a,w (the apparent wall shear rate) then, a viscosity at a particular shear rate may be obtained from the Dynisco Polymer Test LCR7000 capillary rheometer from the expression:

$\eta_a = (\tau / \gamma• a,w) = [(P\ Rc / 2Lc)] / [\ 91.203\ (S / R3\ c)]$

$= [P\ R4\ c\ ] / [182.4\ S\ Lc\ ]$

Viscosity Curves for Some Engineering Thermoplastics

★	PBT (250 C)
+	PPE/PS (270 C)
■	PMMA (230 C)
◆	PA6 (250 C)
●	ABS (217 C)
▲	PC (300 C)

This is an apparent viscosity. In practice the word 'apparent' is often omitted and the symbol η is used. As flow testing is performed over a range of conditions (temperatures and/or piston speeds), the variation of viscosity with temperature and rate of flow can be easily obtained by simple calculations. With the Dynisco Polymer Test rheometers, these calculations are performed automatically by a built-in microprocessor or connected computer and displayed in tabular or graphical form.

Mold Filling Data.

Shear flow data obtained from the capillary rheometer are available in material databases such as those maintained by Moldflow Ltd. The data is used in software designed predict how an injection mold will fill. For this purpose the effect of changing temperature at a constant shear rate is needed, together with values that show the effect of changing shear rate at a constant temperature.

Tabulated Results.

The figures in the following table were obtained when samples of nylon 6 (PA 6), produced by Akzo,

were tested. (Such results are typically obtained using a die with high L/D ratio, for example, 20:1) These figures clearly show that the three, injection-molding grades tested, have very different viscosities, with the first grade having the lowest, that is, it is the easiest flowing grade. It would be used where mold filling is difficult, or where long flow lengths are involved. The viscosities of all three materials fall as the shear rate is increased. Thus mold filling becomes easier at higher rates. Raising the melt temperature, while keeping the shear rate constant, reduces the viscosity and thus the amount of injection pressure required to maintain a certain rate of flow.

MOLD FILLING DATA FOR AKZO 'AKULON' PA 6.

Viscosities (Pa-s) at 1,000 sec^{-1} at the following temperatures:

Grade	°C/°F (240/464)	°C/°F (260/500)	°C/°F (280/536)	°C/°F. (300/572)
Easy flow	98	78	63	50
Medium flow	149	115	89	69
Stiff flow	394	319	259	210

Viscosity (Pa-s) at 280°C/536° F

Grade	100sec^{-1}	1,000sec^{-1}	10,000sec^{-1}	100,000sec^{-1}
Easy flow	197	63	20	6
Medium flow	323	89	25	7
Stiff flow	1220	259	55	12

The Power Law Equation
In practice the slope of the log-log flow curve is not a straight line, as n* varies slightly with shear rate. However, for many purposes it may be assumed to be linear and is equated with a single value of n, the flow behavior index, or the flow index. Using this assumption the Power Law equations for flow are developed:

$\tau = K \dot{\gamma}^n$ or $\eta = K \dot{\gamma}^{n-1}$ (Where n = n*)

This equation works very well with PVC and is often used to describe its flow behavior.

The Power Law

$$\eta_{pl} = m \left| \dot{\gamma} \right|^{n-1}$$

Where: m = Constant
      n = Flow index

n < 1 for shear thinning fluids.

n = 0 for Newtonian fluids.

➤ Describes only the shear thinning portion of the curve.
➤ The value of n ranges from 0.2-0.6 depending upon the type of polymer, MW, and MWD.

The Cross/Carreau Equation

By combining the Newtonian and Power Law Equations with a function to describe the "break" in the viscosity curve (See Figure 13.) Cross and Carreau (Chiang, H., Hieber, C., and Wang, K., SPE ANTEC Technical Papers, Vol. XXXIV, pp.1099-1102 (1988).) have developed and equations that fit many experimental viscosity curves very well. A combined form of these equations is shown in Figure 14. In this equation η_o is the "zero shear rate" or Newtonian viscosity for the particular polymer, n is its Power Law exponent, and τ' is the shear stress at the "break". These equations along with the Power Law and several others are resident in the LabKARS Rheology Software used with the Dynisco Polymer Test LCR Series of capillary rheometers.

13.2 Effect Of Temperature

As can be seen in the data shown above and Figure 13 temperature significantly affects the viscosity of polymers. In general, viscosity decreases as the temperature is increased. Behavior contrary to this observation can usually be attributed to further reaction or cross-linking taking place in the material.
The Ahhrenius Equation
$\eta = k\, e^{(E/RT)}$
where: η = shear viscosity
    k = constant
    E = viscous energy of activation for the polymer (Kj / mole)
    T = the absolute temperature (in oK)
    R = gas constant (0.008314 J / oK mole)
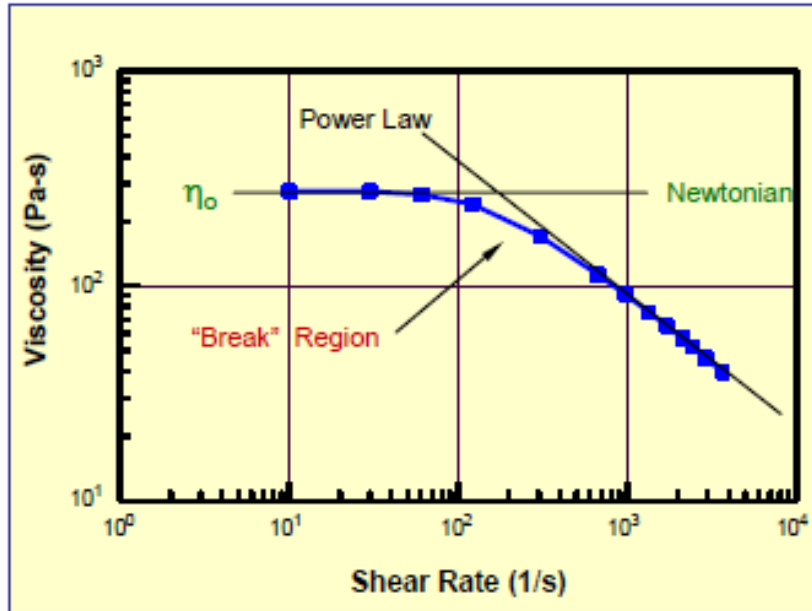
The Viscosity Curve

Figure 13.

The Generalized Cross-Carreau Equation

$$\eta = \frac{\eta_o}{\left[1 + \left(\frac{\eta_o \dot{\gamma}}{\tau^*}\right)^a\right]^{\frac{1-n}{a}}}$$

> Combines the power-law and Newtonian regions of the complete flow curve.
> Useful for numerical simulations that require a more complete description of the flow curve.

n = Flow index
τ^* = Stress at the break in the curve.

$a = 1 - n$ (Cross Model)
$a = 2$ (Carreau Model)

(Hieber & Chang, Cornell Univ.)

is usually used to describe the temperature dependency of polymers at temperatures in excess of 100 oC above their glass transition temperatures (Tg). For most polymers a plot of ln η vs. 1/T yields a straight line over a temperature range of 50 – 150 oC. Thus if the viscosity of a polymer is known at temperature T2 and we can calculate its viscosity at a temperature T2 from the equation ln [η(T1) / η(T2)] = (E/R) [1/T1 – 1/T2] using the activation energies shown in the table below.

EFFECT OF TEMPERATURE

| Polymer	Activation Energy (Kj/mol)	% Change in η/oC	at Temp (oC)
PET	45	1.7	285
HDPE	25	1.4	190
LDPE	49	-	190
Nylon 6	36	1.7	230
PP	44	2.1	230
PC	85	3.1	300
PMMA	65	3.1	230
PVC	85	4.8	190

The table above also shows the percentage of change in the viscosity that occurs when the temperature, which is close to the temperature shown to the right, is increased by 1 oC. In general, the linear form of a polymer has the lower activation energy. As one increases the side chain lengths the activation energies tend to rise. (Note that PVC is very sensitive to temperature.) The calculations shown above assume a constant shear rate. There are different activation energies if the measurements are made at constant shear stress. The relationship between the activation energies at constant rate and constant stress can be shown to be

$E\tau / E\gamma = 1 - \gamma (\Delta\eta/\Delta\tau)T$

Where: $E\tau$ = shear stress dependent energy of activation

$E\gamma$ = shear rate dependent energy of activation.

LLDPE — Apparent Viscosity (Pa-s) vs. Shear Rate (1/s); T= 190 C, T= 230 C, T= 285 C
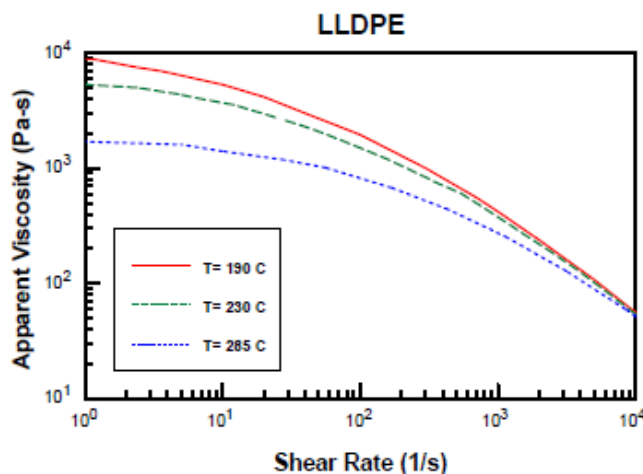
The table below shows a comparison of the activation energies at constant shear rate and constant shear stress for a low-density polyethylene material in the range of 120 –230 oC.

Shear Rate (γ) (sec-1)	Eγ (Kj/mole)	Shear Stress (τ) (Pa)	Eτ (Kj/mole)
0	49	0	49
10-1	43	103	57
100	39	104	68
101	32	105	72
102	27		
103	23		

13.3 Effect Of Pressure

Sometimes it is assumed that molten polymers are non-compressible fluids. It has been shown, however, that due to their molecular structures, they all have a certain amount of "free volume". Under pressure this "free volume" is reduced resulting in a compression of the fluid, which causes an increase in the viscosity. It has been shown that the viscosity increases exponentially with increasing pressure and can be represented by the expression:

$\eta (P) = \eta (P=0) \, e^{\beta P}$

Where P is the pressure and β is the pressure dependence coefficient, which is characteristic of the polymer. Studies with polystyrene show that its viscosity doubles at a pressure of 10,000 psi (69 MPa). Some data for polyethylene are shown below

Polyethylene
($\beta = 0.0033$ atm^{-1})

Pressure (atm)	η / η_o
30	1.11
100	1.39
300	2.70
500	5.29
1000	27.9
3000	22026

13.4 Effect Of Molecular Weight And Molecular Weight Distribution

Of all of the parameters that affect the melt rheology of a particular polymer, the molecular weight (which reflects the size of the molecules) is the most important. Below a critical molecular weight (MC), where molecular entanglements begin to occur, there is a direct relationship between the melt viscosity and the weight average molecular weight (MW). Above MC the "zero shear rate" melt viscosity (where the viscosity becomes Newtonian) for a linear polymer depends on weight average molecular to the 3.4 power. (See Figure 16.) Above the Newtonian region the viscosity is still related to MW, however, the exponent decreases with increasing shear rate. For branched resins a relationship between viscosity and MW exists that includes a constant branching parameter "g" and an exponent of 6.6 to 6.8. Thus as the average molecular weight increases we see an increase in the melt viscosity of a polymeric material. (See Figure 17.) It is interesting to note that while the shear sensitivity of most commercial polymers increases with increasing molecular weight, the temperature sensitivity and flow activation energy are not affected.

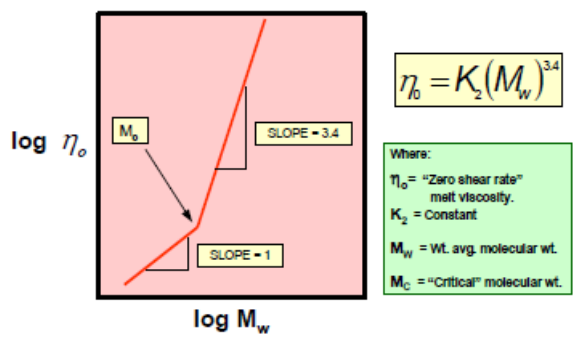**Relationship of the Molecular Weight to Melt Viscosity**

$$\eta_0 = K_2 (M_w)^{3.4}$$

Where:

η_o = "Zero shear rate" melt viscosity.
K_2 = Constant
M_w = Wt. avg. molecular wt.
M_C = "Critical" molecular wt.

log η_o

M_o SLOPE = 3.4

SLOPE = 1

log M_w

Figure 16.

Effect of Molecular Weight on Melt Viscosity

High Average Molecular Weight

log Viscosity

Zero Shear Rate Melt Viscosity (η_o)

Lower Average Molecular Weight

log Shear Rate
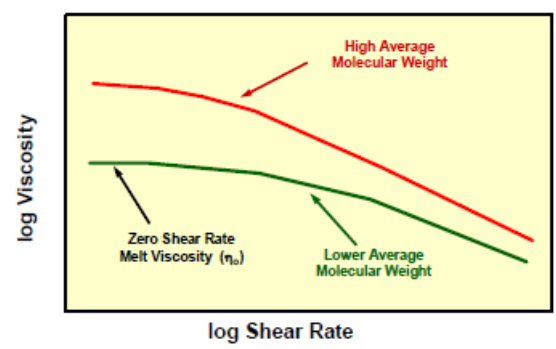
$$MWD = M_W / M_N$$

Where: M_W = weight average molecular weight.
and M_N = number average molecular weight.

A MWD = 1 means that all of the molecules have the same size (or molecular weight) while a MWD > 1 means there is a range in the sizes of the molecules present. As the MWD becomes higher (broader MWD) more high and low molecular weight molecules are present in the polymer and this affects the shear thinning behavior of the melt. As shown in Figure 18 as the MWD broadens the shear thinning behavior begins at a lower shear rate. This often produces better processing conditions for a material, but may also cause a resultant deterioration in physical properties due to the presence of more low molecular weight molecules.

Effect of Molecular Weight Distribution on Melt Viscosity
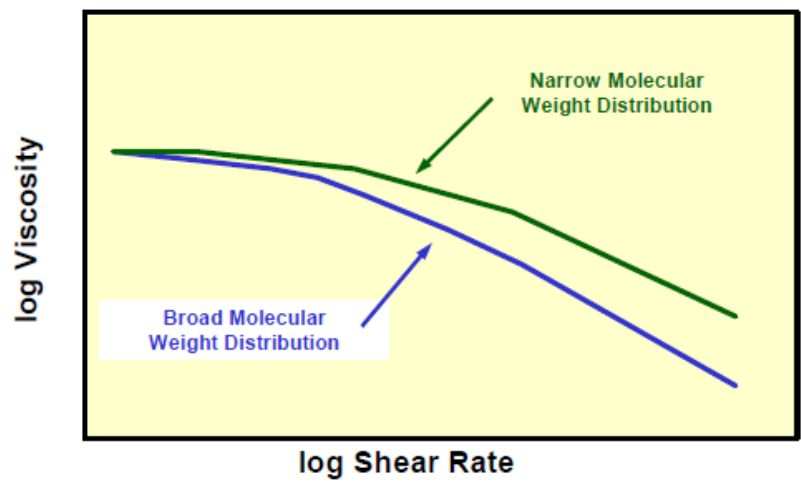

Effect of Various Factors on the Viscocity Curve

Although we have discussed the effect of temperature, pressure, molecular weight and molecular weight distribution on the viscosity curve, there are many other factors that may change its behavior. Some these factors include the presence of other (compatable or incompatable) polymers, fillers, lubricants, or plasticizers. Since most plastics are composed of a blend of a polymer and one or more of these additives, their effect is important to the properties of the molten plastic. The specific effects of these additives can be found in many of the included references in the "For Further Reading" Section. Figure 19 provides a general guide to the effects of some additives, along with temperature,

pressure and molecular weight, on the viscosity curve.

How an <u>Increase</u> in Various Factors Affects the Polymer Melt Viscosity Curve
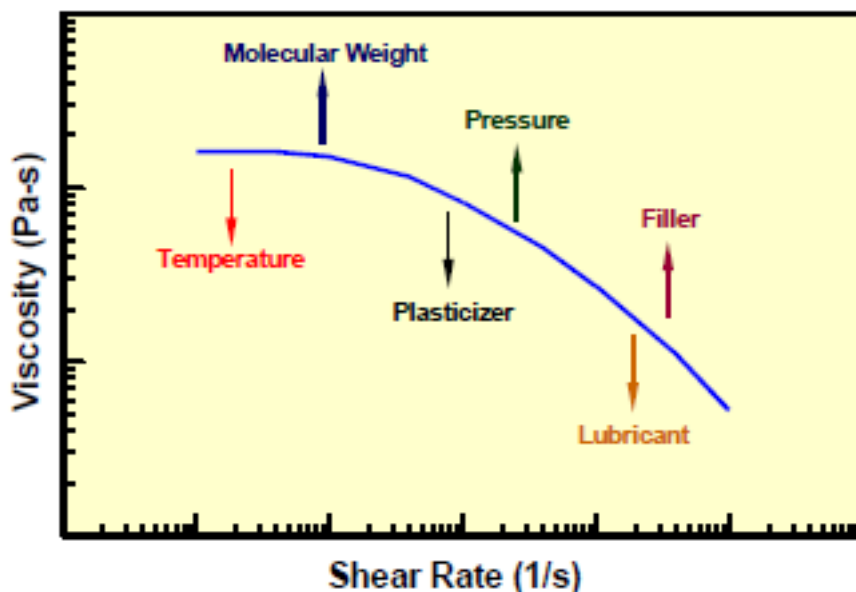
Thermal Stability

All of the discussions above have assumed that the polymer or plastic resin remains unchanged by the thermal conditions present in the test or process. As most polymers are organic (carbon based) compounds they are generally unstable towards exposure to the high temperatures they experience in conversion processes. For this reason the processes are generally designed to expose the polymer to the minimum time (residence time) possible. When a polymer is exposed to an olysulfone high temperature for a short period of time or just a hot temperature for a long time it can be affected in one of two ways. Chemical bonds in the molecules can be broken to produce smaller molecules or the polymer can continue to react with other molecules or may crosslink to form larger molecules. (See Figure 20.) The former behavior can be seen in polymers such as PET, the nylons, and PMMA, while PVC provides a good example of the latter. In either case the melt viscosity will be affected. As shown in the earlier section, the molecular weight of a polymer has a very string effect on its melt viscosity. Thus degradation of the polymer chain will be seen as a decrease in the viscosity, while continued polymerization or crosslinking will be reflected in an increase. (See Figure 21.) In addition to affecting the melt viscosity of the resin, these processes invariably degrade the final mechanical properties of the finished part.
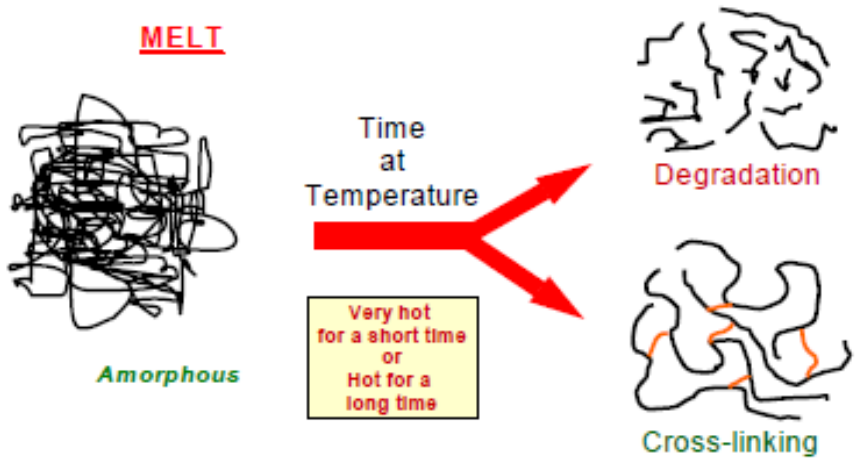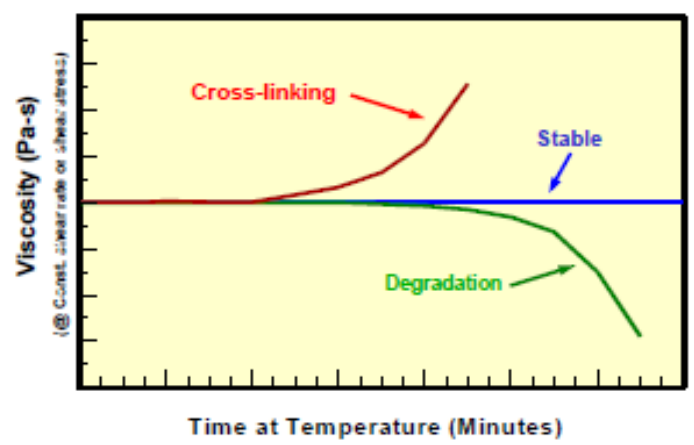
Figure 20.

As a result of the effect of temperature on the properties of a finished part, it is important to obtain a measure of the thermal stability of polymer or plastic resin at the temperatures used in the process. As the viscosity is very responsive to changes in the molecular weight of the polymer, it provides a sensitive measure its thermal stability.The

rate at which degradation or crosslinking occurs in a polymer or plastic, at a given temperature, can easily be measured on a capillary rheometer. (See Figure 21.) In this test, sometimes called a "time sweep", the rheometer is set up to perform measurements at one shear rate (or stress) at the set temperature. (It is best to use a moderately low shear rate to obtain the highest sensitivity.) When the test is initiated a first measurement of viscosity is acquired. The rheometer is then stopped for a pre-set period of time, after which another measurement is made. This procedure, which is usually controlled by a computer, is repeated until a pre-set total test time is reached. This test can then be repeated at a series of different temperatures. When the viscosity verses time data are fit to a line, one can obtain a measure of the rate of degradation or cross-linking. As an example of the data obtained from this test, Figure 22. shows the results from a series of "time sweeps" for Nylon 6/6 at a series of different temperatures. These data can then be used to set a "processing window" for this resin. (A similar test can be carried out using the MFR Tester – See the segment on MFR Testing in the Applications Section.)
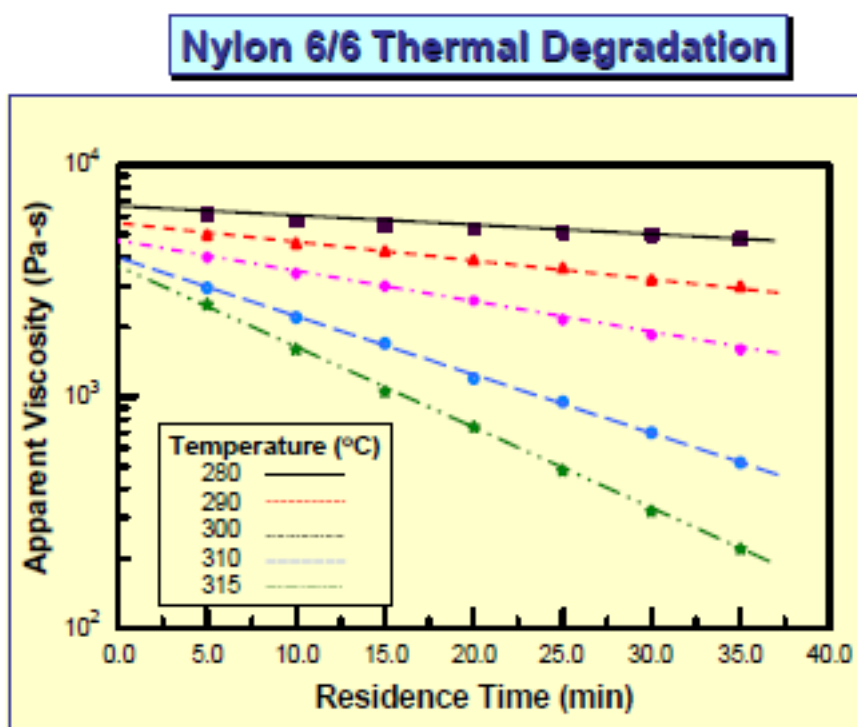
Bagley, Rabinowitsch And Head Corrections

The flow curve that is most commonly produced from capillary rheometry data is one in which the shear stress at the capillary wall (τw) is plotted against the apparent shear rate at the capillary wall ($\bullet\gamma$ a,w). These two material properties are obtained using the formulae:

$\tau w = \Delta PR_c / 2L_c$ and $\dot{\gamma}_{a,w} = 4Q/\pi R^3 c$

It is also common practice to define apparent viscosity (η_a) by the equation

$\eta_a = \tau w / \dot{\gamma}_{a,w}$ (The symbol μ_a is sometimes used instead of η_a.)

The Rabinowitsch Correction

The Rabinowitsch Correction is a correction applied to viscosities, obtained from a capillary rheometer, to rectify the data for the non-Newtonian character of that fluid. For Newtonian fluids the apparent shear rate at the wall is equal to the true shear rate ($\dot{\gamma}w$).

However, for non-Newtonian fluids, such as polymer melts, the two are not equal but are related by the equation

$w\dot{\gamma} = [(3n*+1)/4n*]\ \dot{\gamma}_{a,w}$

Where $n* = [\Delta log(R\Delta P/2L)]/[\Delta\ log(4Q/\pi R)^3]$

This equation is referred to as the Rabinowitsch Equation and the term in square brackets is known as the Rabinowitsch Correction. While in theory the magnitude of the correction could range from 1 to ∞, the maximum error in the viscosity at any shear rate has been shown to be 15% when $n*$ is 0.23. Generally, it is less than 15% when dies of L/D of 20:1, or greater, are used.

Approximate Method (Macosko, pp 242-4)

When one compares the true shear rate at the wall and the apparent shear rate, it can be shown that they are equal near the wall. (See Figure 15) It has been determined that this occurs at a point that is 83% of the capillary radius. Thus an approximate value of the shear rate at the wall can be found from the relationship $\dot{\gamma}w = (0.83)\ \dot{\gamma}_{a,w}$

It has been shown that this approximation produces an error of less than 2% for a range of n values from 0.2 to 1.3. The main danger in using this approximation is when the fluid is very strongly shear thinning (n ≤ 0.1), where significant error occurs. Correction: Use and Application. It should be borne in mind that:

1. For comparative purposes applying the correction will not alter the comparability of the data.
2. For a tubular die the relationship between shear stress at wall and the apparent shear rate at wall is unique and does not depend on the size of the tube or capillary.
3. The apparent flow curve ($\dot{\gamma}_{a,w}$ vs. τ) can be used for calculating data for flow through slits with a maximum error of as little as 3%, which is within the accuracy of most measurements.

For these reasons, the only occasion where it is necessary to use the correction is for research or for design purposes, where it is required that the true shear rate be known. For practical scale up purposes it is not important. A knowledge of $n*$ is useful, however, as a measure of the degree of non-Newtonian behavior of the melt. The lower the value of $n*$, the more non-Newtonian the melt

The Rabinowitsch-Mooney Correction

Corrects shear rate at the wall for non-Newtonian fluids.

$$\dot{\gamma}_w = \left[\frac{3n+1}{4n}\right]\left(\frac{4Q}{\pi R^3}\right) = \left[\frac{3n+1}{4n}\right]\dot{\gamma}_{w,a}$$

Where:

$$n = \frac{d\ln(R\Delta P/2L)}{d\ln(4Q/\pi R^3)}$$

= "Flow Behavior Index" (Power Law)
(= Slope of τ_w vs. $\dot{\gamma}$ - Varies with shear rate.)

$\dot{\gamma}_{w,a}$ = Apparent wall shear rate

Effect of Flow Index on Shear Rate

Flow Index (n)	(3n + 1) / 4n
1.00	1.00
0.75	1.08
0.67	1.12
0.50	1.25
0.33	1.51
0.25	1.75
0.10	3.25

The Bagley Correction

The Bagley Correction, also known as The Bagley Entrance Correction, is possibly of greater practical importance than the Rabinowitsch correction. There is an incorrect assumption in the basic analysis for flow through a tube that all of the energy involved in forcing the material through a tube is absorbed over the length of the tube. In reality, some energy is used in funneling molten polymer molecules from the barrel or reservoir into the die. Much of this energy absorption is due to that involved in deforming the polymer molecules as they are suddenly subjected to an increase in elongation just above the die. This required extra energy input appears as equivalent to an increase in the effective length of the die.
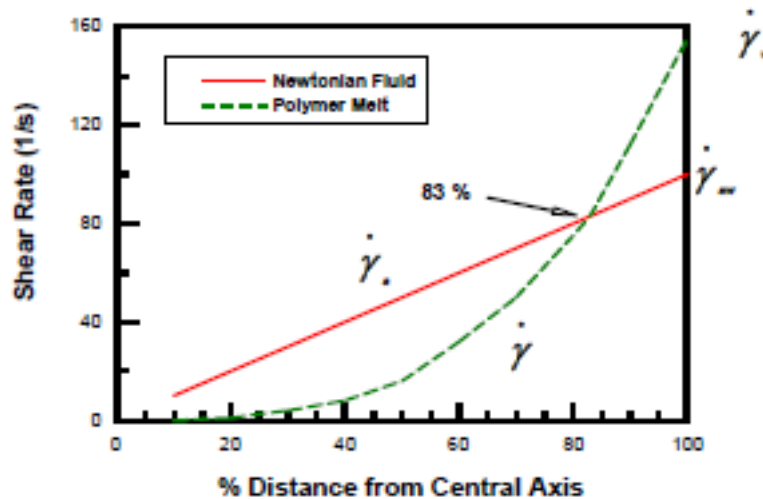
Bagley measured the pressure drop (ΔP) against •γ w,a using a series of dies with the same diameter, but different L/D ratios. The subsequently derived plots of ΔP against L/D at constant shear rate showed an effective increase (eo) in length/diameter ratio, which can be obtained by reading off the negative intercept of the ΔP-vs.- •γ w,a plot. In this case the true shear stress is given by τtrue = ΔP/[2(L/R) + eo]

Some measure of the error involved in not considering the Bagley correction may be gauged by considering a tube with a L/R ratio of 10/1 and a polymer melt with a Bagley correction factor (eo) of 5. Ignoring the correction would give a wall shear stress of ΔP/20 whereas using the correction will give a value of ΔP/30, a 50% error!

Alternatively, one can determine the value of ΔP at each shear rate when the line in the ΔP-vs.- •γ w,a plot reaches L/R = 0. This value, designated as ΔPo, represents the pressure drop for a "zero length" die. Since the die has zero length we can assume that no shearing takes place and the entire pressure drop is due to the extension of the polymer molecules. (Cogswell has used this assumption to develop a method and equations for the measurement of extensional viscosities with the capillary rheometer. If one uses a "zero length" or very low L/D die it is possible to easily obtain relative values of the extensional viscosities of different polymers.) Thus if we subtract this pressure from the total measured pressure, we can calculate the true shear stress.
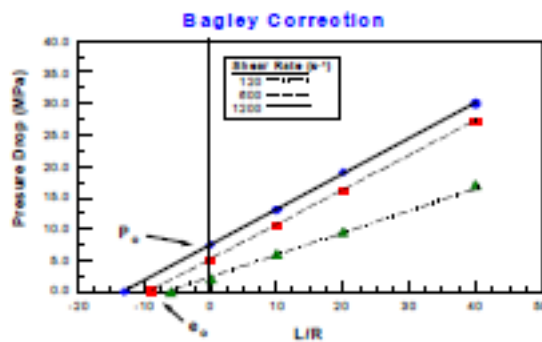
$$\tau_{true} = (\Delta P - \Delta P_o)\,/2(L/R)$$

The Bagley Correction

Corrects shear stress for entrance pressure drop.

$$\tau_w = \frac{(\Delta P)R}{2(L + e_o R)} = \frac{\Delta P}{2(L/R) + e_o} = \frac{\Delta P - P_o}{2(L/R)}$$

Where:

e_o = End Correction Factor
P_o = Pressure Drop
at L/D = 0

Bagley Correction

Yet another method of correcting for the entrance effect, similar to that discussed above, is that known as the Couette-Hagenbach method. This is a 'difference' method in which two dies of different lengths L1 and L2 are used and for each of which the output Q for a series of pressure drops ΔP are obtained. Since for each die at a given shear rate the entrance effect is the same, the pressure drop and length of the shorter die can be subtracted from those of the longer die to give an estimate of the pressure drop ΔP1-ΔP2 for a hypothetical die of length L1 – L2. From this, the true shear stress at wall may thus be obtained from the formula

$$\tau w = (\Delta P1 - \Delta P2) / 2(L1 - L2)$$

While this approach does correct for the entrance effects, it gives no idea of the magnitude of the error, which may be of interest in scale-up.

Head Corrections.

In some rheometers, and particularly those used for measuring flow rate (MFR or MFI), the force used to push melt through the die is measured above the melt in the reservoir (usually by measuring the weight or force put onto the melt.) There will be some error in the flow curve calculation if pressure losses in the reservoir are ignored. It may be shown that when the length of melt in a reservoir of radius Rb is Lb and the length and radius of the capillary are Lc and Rc, respectively, the ratio of the pressure drop in the reservoir or barrel (ΔPb) to the pressure drop in the capillary or die (ΔPc), for a true power law fluid, is given by:

$$\Delta P_b / \Delta P_c = (L_b / L_c) (R_c / R_b)^{(3n^*+1)}$$

In the determination of melt flow rate, most standards now specify that measurements must be made with the piston within fairly closely defined limits above the die. This is done to assure that, in all measurements, the amounts of material in the reservoir are similar.

Combined Rabinowitsch and Bagley Corrections Of Shear Rate and Shear Stress
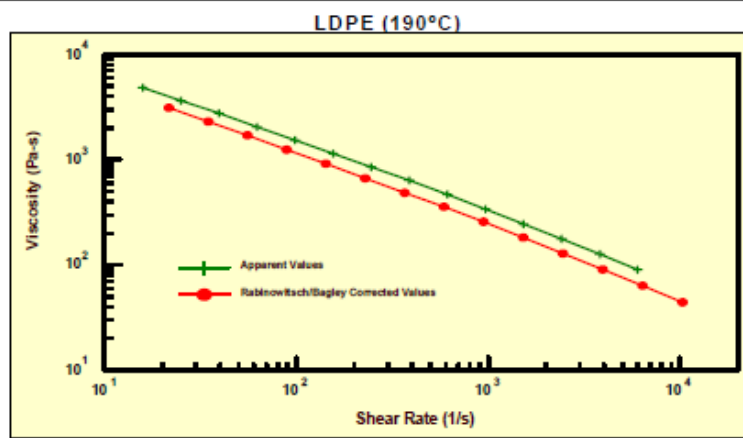
Wall Slip.

Flow analysis generally assumes that there is no slip of the melt at the wall of the capillary. However, this assumption is not necessarily true (particularly at high shear rates.). If such slip exists, then a form of plug flow is superimposed on the normal laminar flow pattern. It has been shown [Lupton and Regester, Polymer Engineering & Science 5 235 (1965)] that if the wall slip velocity is Va, Q is the volumetric flow rate, and Rc is the capillary radius, then:

$$4Q/\pi R^3c = 4Va\,(1/Rc) + X\,)$$

where X is a function of the shear stress.

To obtain a correction for this a series of flow curves is produced using a set of dies of varying radius Rc. Then, using measuring $4Q/\pi R^3$ at a given value of shear stress, a plot may then be obtained of $4Q/\pi R^3c$-vs-$1/Rc$. The slip velocity will then be one quarter of the slope of this plot. This may then be repeated at other shear stresses and this will enable a plot of slip velocity against shear stress to be developed.

MEASUREMENT OF EXTRUDATE SWELL

Die swell is also known as die swell, swelling ratio or puff-up ratio and is defined as the ratio of the diameter of the extrudate to the diameter of the die.

Errors due to drawing down. Accurate measurement, and interpretation of extrudate swell requires care. When it is measured on samples extruded downwards from a capillary rheometer the weight of the extrudate may to cause drawing down (stretching). Drawing down can be avoided by extruding into a bath, containing a liquid of the same density as the polymer, located immediately below the die. The effect of draw down, however, may be more conveniently minimized in other ways. Minimization of errors. Draw down may be minimized graphically. Measurements of extrudate diameter are made at several points away from the leading end of the extrudate (obtained when cutting the extruded sample at the die outlet). By means of a graph, the theoretical die swell at this leading edge may be found by extrapolation. There is little loss in accuracy if the experimenter always makes the measurements at the same distance from the leading edge. The distance chosen is that at which the extrudate swell is approximately a maximum. Circular cross-section. To ensure that a circular cross-section is obtained from the die or capillary, the extrudate must be given time to freeze while suspended since it can distort if allowed to lie on a surface before it has hardened. This distortion may be a problem when extruding unvulcanized rubber compounds, as extrudates made from such soft materials may distort under their own weight even when cold.

Diameter Determination.

Providing the extrudate is smooth and circular, the measurements of diameter may be made using a projection microscope. If a laser micrometer is available, very accurate measurements (of the extrudate and of the die) can be made relatively easily. Such data may be logged into the computer of interfaced to the rheometer. (This is the method used with the Dynisco Polymer Test LCR 7001.) If the sample is

not round and smooth or if the extrusion exhibits sharkskin or melt fracture the best procedure is to weigh (M) a known length (L) of the extrudate. From these data and knowledge of the density (ρ), the extrudate diameter (D) can be calculated from the equation:

$$D^2 = 4M/\pi\rho L$$

Since it is usually required to know the ratio of the product diameter/die diameter, it is satisfactory to use the density of the solid polymer. In the case of crystalline polymers, however, the density will depend on the rate of cooling from the die. For accurate measurement the density should be obtained from the extruded sample.

Measurement of Extrudate Swell
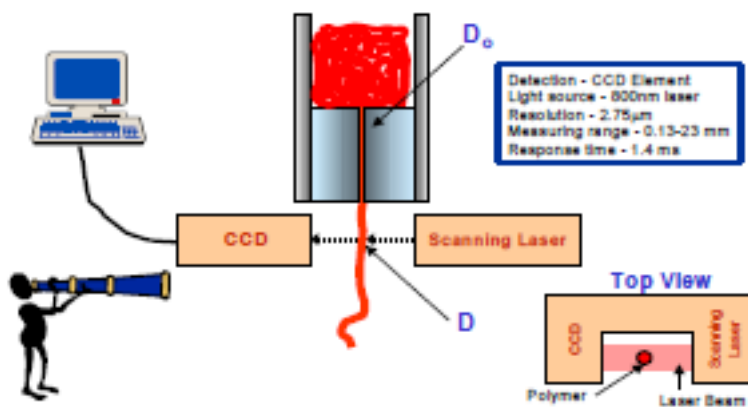
Errors due to molecular factors. For semi-crystalline thermoplastics, estimates for the extrudate swell of large diameter slow cooling industrial extrudates, obtained from fast cooling small laboratory samples, may show some error. A source of this error may arise from the freezing of the extrudate before the polymer molecules have had a chance to recoil or relax after emergence from the die. This is a greater problem with small diameter extrudates and thus a source of scale up error. Error in the laboratory samples may be reduced by allowing them to anneal at a temperature 50-60ºC, above the glass transition temperature. On occasion it may be desired to know the ratio of the melt diameter/die diameter. In this case the melt density must be used in the formula above. When projection microscopy is used, the measured value of the diameter of the solid extrudate should be multiplied by the ratio solid density/melt density.

## Viscosity and Extrudate Swell as a Function of Shear Rate
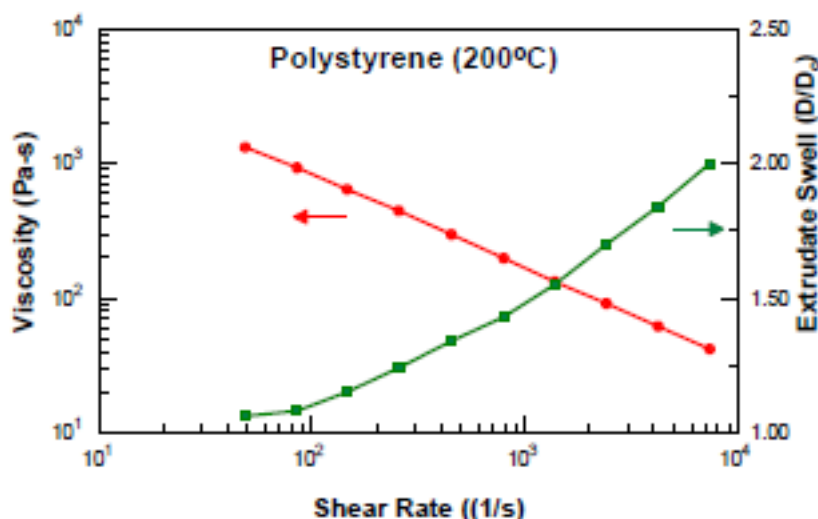
APPLICATIONS OF RHEOLOGICAL DATA

Novel Applications Of The MFR Tester

The ability of the standard MFR instrument to characterize polymers is considered by some to be rather limited. However, the capabilities of this instrument for polymer testing go well beyond those achievable strictly by the standard tests. Several of these applications are discussed below. Shear Thinning of the Melt – The MFR test is sometimes criticized for being a single point test. Because of their nature, the flow properties of polymers usually vary with the force or rate at which they are deformed. As a result of the orientation of the polymer molecules, their flow rate often increases faster than expected as the deformation rate or force is increased. Therefore, a function rather than a point will best describe their flow behavior. This behavior can be gauged with the MFR tester by measuring the melt flow rates generated using a series of two or more different weights. The slope of the curve obtained by plotting the weight verses the MFR will give a measure of this "shear thinning" function [See Yoo, H.J., et. al., SPE ANTEC, Technical Papers, Vol. XXXIX, pp. 1444-47 (1993)]. For some polymers this slope can be roughly related to its molecular weight distribution. (See Figure 25.) Note in the figure that resins PE-2 and PE-5 have the same MFR in the standard low load (2.16 Kg) test, but are very different at the higher loads. Thus, whereas the standard MFR test would indicate that the resins are the same, they are not.

Multi-Weight MFR Test

Legend: PE-1, PE-2, PE-5

Y-axis: MFR (g/10 min.)
X-axis: Weight (Kg)
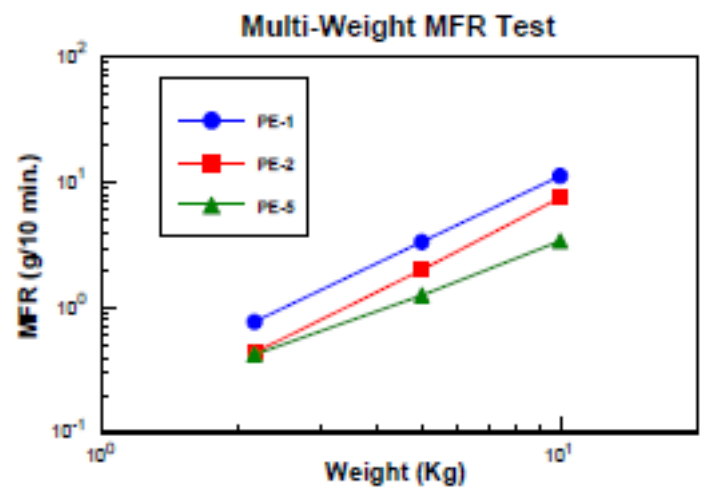
Extrudate Swell – When polymer melt emerges from a die the extrudate may swell so that its cross-section, after it leaves the die, is significantly greater than that of the die orifice. This source of this swelling is from the extension or, orientation, of polymer molecules as they pass through a die. On exiting the die the molecules tend to re-coil back to their original configuration, which causes the extrudate to expand perpendicular to the flow. The extrudate swell is defined as the ratio of the extrudate diameter to that of the die. This value is very important to anyone extruding a profile, where dimensions are critical.

The melt index test itself will not give a measure of extrudate swell. However, a semi-quantitative measure of the relative swell values for a product can be obtained from measurement of the length the polymer strands produced during equal extrusion times. The shorter the strand the greater will be the extrudate swell. All other things being equal, a material that swells more is more "elastic" and will show less sag in thermoforming operations.

Thermal Stability - PVC

180 C
220 C (Cross-linking)

Y-axis: MFR (g/10 min.)
X-axis: Time (Min.)
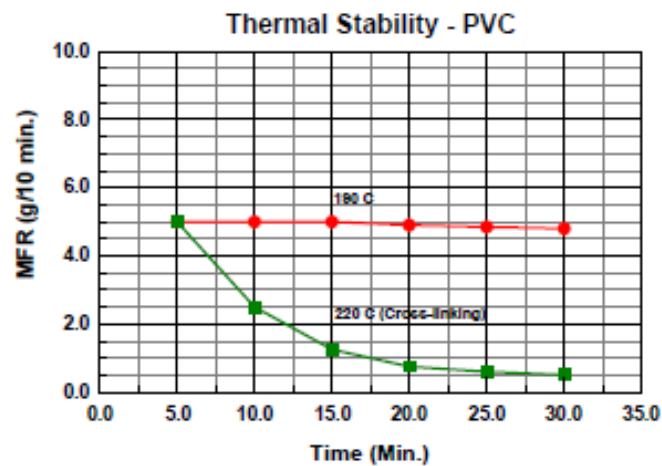
Thermal stability – The stability of a polymer product at processing temperatures will often determine if a material can be successfully formed into useful products that don't fail in service. While the melt indexer is often used to determine if a material has degraded, after the fact, it can also be used to determine if a virgin material has the stability required to stand up to a process. To do this one need only set up the MFR test by the standard procedures. Then a separate timer should be started once the barrel is filled and the piston inserted. Next conduct a standard Method A or B test noting the MFR and the completion time from the timer. While supporting the weight and piston, pause for a period of time, say five or ten minutes and then, run another test and record the MFR and time. This procedure can be repeated several times. The results, plotted as MFR verses time at temperature, will then reflect any changes taking place in the resin and the rate at which they occur. An increase in MFR will indicate that the polymers are degrading, while a decrease will indicate the possibility of cross-linking taking place (See Figure 26.). This test can be repeated at other temperatures to establish the "processing window" of a material.

Intrinsic or Relative Viscosities – Intrinsic or relative solution viscosity measurements have traditionally been used to estimate the molecular weights of polymers. They are extensively used in the manufacture of PET, nylons, and other polymers. These tests, however, are time consuming, require laboratory facilities, and necessitate the disposal of the frequently unfriendly solvents used. The melt viscosity, and therefore the MFR of a polymer, is also directly related to its molecular weight. (In fact, it is five times more sensitive than the solution viscosities.) Thus a relationship exists between the two methods and MFR measurements can be used to determine solution viscosities. Figure 27 shows that a semi-log plot of the MFR of several intrinsic viscosity standards produces a straight line. Thus once this relationship is established the IV can be determined from MFR in a short time and without the use of unfriendly solvents. [See Reilly, J.F. and Limbach, SPE ANTEC, Proceedings Vol. XXXVIII, pp 812 – 15(1992)].

MFR vs. IV
PET @ 285 °C

Tests Performed by ASTM D3835 and D1238
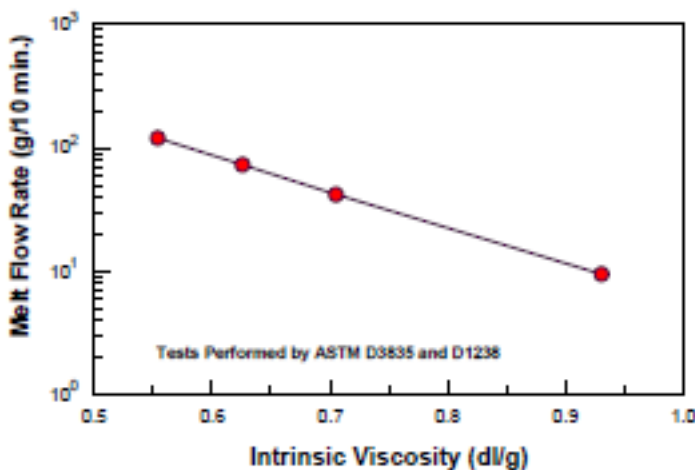
Correlation Of Polymer Melt Rheology To MFR

A correlation of the shear flow based melt rheology to melt flow index (MFR, g/10min) or melt volume index (MVR, cc/10min) is made by the following procedure: 1.) Match the shear stress achieved in the melt indexer on shear stress verses shear rate curve. 2.) Determine the shear rate achieved at this shear stress. 3.) Calculate the volumetric flow rate from the shear rate and die geometry.

Method

The MFI test is performed under a condition of constant stress. The stress exerted upon a molten polymer in the melt flow indexer is calculated from the weight of the piston and standard weight, piston face area, and the die geometry. A table of these stress values for the ASTM D-1238 test specification is shown below. (Similar values can be calculated for the ISO 1133 test.)

Shear Stresses in ASTM D-1238 Tests

Test Load (weight + piston) (g)	Shear Stress (psi)	Shear Stress (kPa)
325	0.4256	2.932
1050	1.3750	9.474
1200	1.5714	10.827
2160	2.8318	19.511
3800	4.9762	34.286
5000	6.5476	45.113
10000	13.0952	90.226
12500	16.3690	114.256
21600	28.318	195.11

A laboratory capillary rheometer can be used to obtain a measure of the MFI by a determining of the volumetric melt flow rate from a curve of shear stress verses shear rate, at constant temperature. Such a curve is shown below. As shown in Figure 28 below, a horizontal line at the shear stress in the MFI test is extended to the curve. The corresponding shear rate is determined from a vertical line dropped to the shear rate axis. A second order polynomial fit of the rheometer data is used in the LabKARS Software to interpolate these values. (A Rabinowitsch Correction is performed on the data prior to interpolation.) The resultant shear rate is used in the volume flow rate equation, with the dimensions of the melt flow indexer die, to determine the flow rate in the indexer at that shear rate.

$$Volumetric\ Flow\ Rate = Q = \frac{600\,\dot{\gamma}\,\pi\,R_c^3}{4} = 0.542\,\dot{\gamma}$$

Where MFI die radius in cm (0.20955/2) and the constant 600 converts from units of 1/seconds to 1/ 10 minutes. (From the above it can be seen that to a first approximation the volumetric flow rate is about one half the shear rate at the melt flow indexer stress level.) The above equation produces the melt volume index (MVI) in cc/10 minutes. The melt flow index (MFI) in g/10 minutes is obtained when the MVI is multiplied by the polymer melt density at the test temperature. Note: Flow in the short (L/D=3.818) melt flow indexer die is not all shear flow. When extensional effects produce a significant fraction of the pressure drop, the predicted MFI will tend to be higher, since extensional effects are ignored in the calculation. If, however, two rheometer dies are used the entrance effect can be estimated and a more accurate MFI produced.

Shear Stress vs Shear Rate
(Log-Log Plot)
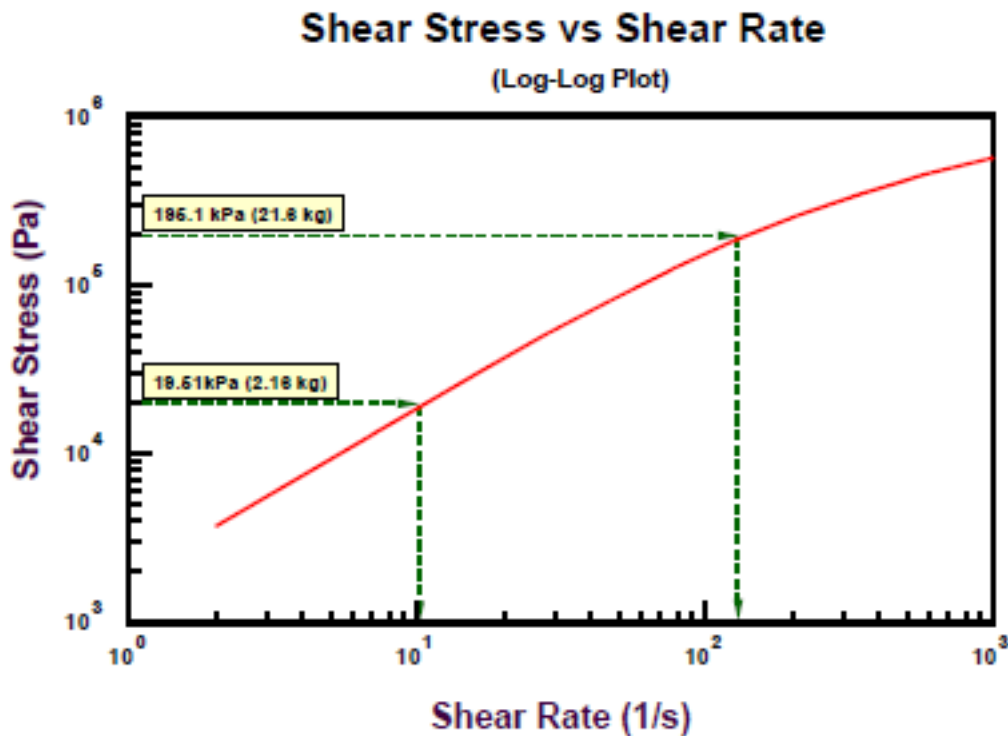
Applications In Extrusion Pressure Drop In A Die

Pressure is required to force a liquid, such as a polymer melt, through an extrusion die. In the case of extrusion dies this pressure is usually produced by an extruder screw that pumps material to the die face where pressure is built up by the restricting effect of the die.

Simple Die Characteristics

In the case of a simple extrusion die of any shape, where the cross-section of the channel is constant from its inlet to exit:

1. The pressure is greatest at the die entry.
2. The pressure is virtually zero at the die exit (and for most purposes may be equated with zero).
3. The pressure drop from the inlet to the exit is linear.
4. When the inlet pressure is denoted by ΔP and the length of the die is L, ΔP/L is constant along the whole length of the tube.

Calculation Of Pressure Drop In A Capillary Die

The pressure drop in a constant cross-section die (which is fed from a reservoir of material at the die head) is related to the output rate, the die dimensions, and the melt viscosity. For a simple rod die, the pressure drop is calculated by the following procedure:
1. Determine the apparent wall shear rate from the formula $\dot{\gamma}_{w,a} = 4Q/\pi R^3$.
2. Read off the wall shear stress (τ_w) corresponding to this shear rate value from the flow curve for the appropriate polymer at the extrusion temperature.
3. Calculate the pressure drop (i.e., the pressure developed at the back of the die parallel) from the shear stress by the formula $P = 2L (\tau_w) / R$ where L is the length and R the radius of the parallel portion of the die.
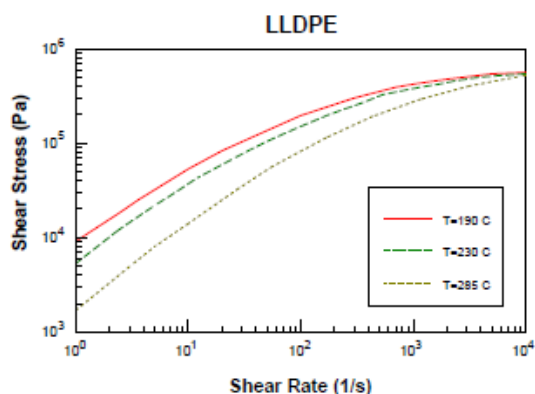
Calculation Of Pressure Drop In A Slit.

For a slit die the shear rate is given by $\dot{\gamma}_{a,w} = 6Q/Wh^2$ where w is the width of the slit and H its thickness (or depth). The flow curve, even though it may have been prepared from capillary data, may then be used to read off the corresponding shear stress. In turn the pressure drop is obtained from the expression
$P = 2L \tau_w / H$

Calculation Of Pressure Drop In A Tapered Capillary Die

In the case of a tapered die, the radius at the inlet is called R1 and at the outlet R2. As with the capillary die, the shear rate is first determined applying the capillary formula using the inlet radius. The shear stress at that shear rate is determined from the flow curve. The pressure drop is then given by:
$P = [2 \tau_1 / 3\tan\theta] / [R1 / R2)^3 - 1]$.

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Sizing Of Die Components

The cross-section of an extrudate will not always be the same as the outlet crosssection of the die from which it has been extruded. There are three reasons for this:

1. Shrinkage of the extrudate on cooling.
2. Extrudate or die swell.
3. Draw down (stretching of the extrudate on emergence from the die).

During rheology testing, die swell measurements can be made relatively easily. A common use of such measurements is to size die components. The magnitude of the shrinkage is usually quite small and since die swell measurements are usually based on cold extrudates this factor is usually built into any die swell assessment. Whereas die swell increases the cross section, drawing down reduces it.

Correction Factors.

The relationship between the cross-section dimension of an extrudate E and the cross-sectional dimension of a die D, is:

E = sdD.

In this equation, s is the die swell ratio at the wall shear rate occurring during extrusion. D is the draw down factor defined by $\sqrt{(L_d - L_u)}$. L_d is the length of material produced in a unit of time in the process in which draw down has taken place and L_u is the length of material naturally extruded in unit time without any drawing down having occurred. (The square root term arises since it is the cross-sectional area rather than a single dimension e.g. diameter that decreases inversely with the extent of draw-down.)

Sizing Pipes.

A further complication arises in pipe manufacture, where it is common practice to inflate the tube to the inner diameter of a sizing die. In this case, if the outer diameter of the extrusion die annulus is do and that of the inner diameter of the sizing die annulus is ds, then the thickness of the pipe T will be related to the annular gap of the die, A, by the relationship:

T = s$\sqrt{(L_d / L_u)}$ (ds / do)A

Estimation of Pressure Drops in Extrusion Dies

The relationships between pressure drop (P), output rate (the flow rate Q), and viscosity for various flow cross-sections may all be arranged to take the form

P = η x R x Q.

Where R is called the die resistance.

Die Resistance.

Die resistance is a measure of the resistance of a flow channel to the passage of the polymer melt. For regular channels of simple shape it is easily calculated from their dimensions. For regular extrudate sections include rod (cylinder), sheet (rectangle), and pipe (annulus), the formulae for calculating the die resistance are:

1. Rod or Cylindrical Channel.

Length = L; Radius = r; Diameter = d.

Die resistance = R = 8 (L) / π r4 = 128 (L) / π d4

2. Rectangular Channel.

Length = L; Width = w; Thickness = t.

When the width is more than 10 times thickness

Die resistance = R = 121 / w t³

b.) Where width is less than 10 times thickness

Die resistance = R = 121S / w t³

S = Shape factor and is given in Figure 3.

3. Pipe or Annular Channel.

Length = 1; inner diameter =dI, outer diameter = do, Gap thickness, t = 1/2(do-di)

Average diameter, d = 1/2(do + di)

Die resistance = R = 121 / π d t³

Calculating Die Resistance (Example For Pipe Die Land.)

Consider a land of length 50 mm, with an internal diameter of 100 mm, and external diameter of 104 mm. The average diameter is 102 mm and the gap thickness is 2 mm.

Using the formula given above:

Die resistance = R = 12.50 / π (102.2)³ = 0.234 mm-³.

Flow Rate

When calculating pressure losses, flow rates must be expressed as volume flow per unit time. If the mass flow rate, for a melt of density D at the processing temperature, is known as M (kg/h) then the volume flow rate (Q) in mm³/s is:

Q = 278,000 (M / D).

If a melt flow rate tester that can do a method B test is available the melt density (D) may be measured directly.

Correction Method For Plastic Melts

The fact that molten plastics are non-Newtonian introduces two complications in calculating pressure losses. First, the viscosity of molten plastic is not constant and second, the flow pattern in a channel is different from that of a simple Newtonian fluid. (The viscosity of a simple, or Newtonian fluid is a constant, which depends only upon the temperature.) These problems can be overcome by using apparent viscosities and representative values.

Apparent Viscosity.

If the shear stress at a particular shear rate is known for a plastic melt, then the ratio shear stress/shear rate is called the apparent viscosity. The apparent viscosity generally decreases as shear rate increases. To calculate pressure losses in a process it is necessary to have a graph of apparent viscosity against shear

rate for the material at a particular temperature of the melt. These flow curves can be obtained from material suppliers, measured in the laboratory (see Figures 10 and 11 and the section on 'Rheology Equipment and Testing'), or it may be calculated approximately from the MFR of the plastics material. Figure 15 shows a flow curve that will be used as an example here.

Representative Values.

At a given overall output rate, the flow pattern of a molten polymer in a channel will differ from that of a simple Newtonian fluid. Figure 31 shows flow patterns through a 20mm diameter tube for a Newtonian and for a non-Newtonian fluid (polymer melt). The polymer melt is assumed to have a melt flow index (MFR) value of 0.3. Both materials have zero flow velocity at the walls of the tube and maximum flow velocity in the center of the tube. For the same output rate (and hence the same average flow velocity) the polymer melt has a lower maximum velocity and more of a plug like flow. The flow velocity is nearly constant across the middle of the tube and decreases rapidly near the walls. As a result, the shear rate, which controls the localized rate at which layers flow over each other, is greater near the walls in the case of the polymer melt than for a Newtonian. Near the walls of the flow channel there can be a large difference in shear rates as shown in Figure 30.

There is a fairly complex way of relating Newtonian and non-Newtonian shear rates at the wall called the Rabinowitsch correction. However, it is simpler to consider the situation where the two lines in Figure 31 intersect. Here Newtonian and non-Newtonian shear rates are equal. The value of shear rate at this point is called the representative value. By using this value it is possible to calculate pressure drops using a method similar to that used for Newtonian fluids (see W. Michaeli in his book 'Extrusion Dies').

Representative Shear Rate

It has been found that the position of the representative shear rate in a flow channel is nearly constant for plastic melts. For a tube, it is at 83% of the distance from center to walls. The formulae for calculating the representative shear rates are as follows:

For a rod die: $\qquad \dot{\gamma}^* = 3.3Q/\pi\ r^3$

For a rectangular channel: $\qquad \dot{\gamma}^* = 4.6\ Q/b\ t^2$

For an annulus: $\qquad \dot{\gamma}^* = 1.46\ Q/d\ t^2$

Flow Through a Die
for a Newtonian Fluid and a Non-Newtonian
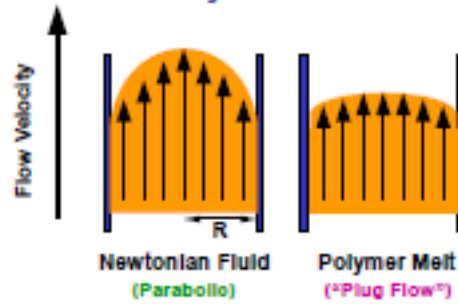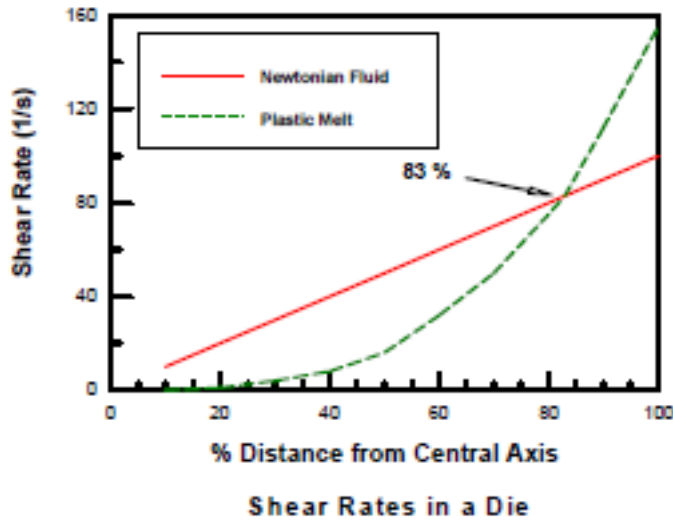Polymer Melt

Figure 34.

Shear Rates in a Die

Representative Viscosity

Once the representative shear rate is determined, the flow curve for the plastic melt is used to find the apparent viscosity of the melt at this shear rate. This apparent viscosity is called the representative viscosity (ηR) and if substituted in the equation **P = ηR x R x T**, it may be used to calculate pressure drops

Pressure Drop Calculation Using Representative Viscosity

Suppose the pipe die whose die resistance was calculated above, extrudes plastic melt at 180 kg/h, the melt has a density of 800 kg/m³, and its flow properties are as shown in Figure 15.
Flow rate: Q = 278,000 x 180/800 = 62,550 mm³/s.
Representative shear rate: * = 1.46 x 62550 / •γ 102 x 2² = 224 s-1
From the flow curve, the representative is viscosity (@ 230 oC in Fig. 19) is 940 Pa-s
(N s / m²).
Pressure drop: P = ηR x R x Q = 940 x 0.234 x 62550 = 13760000 N/m² = 13.76 MN/m² or 2005 psi.

Flow Resistance Of Complete Dies

Most dies may be considered as a number of connecting sections. The die resistance depends most on the smallest dimension of the channel cross-section (for example, the thickness of a slot die or on the gap thickness of a pipe die). Thus flow resistance is usually greatest across the die land and one should first calculate the die resistance and pressure drop of this section of the die. Only when this is satisfactory is it then necessary to calculate pressure drops for the other die sections. The total pressure drop will then be the sum of the pressure drops for each section.

15.4 Applications In Injection Molding.

Pressure Drop in a Runner and Runner Sizing

A runner is the channel that connects an injection mold cavity to the sprue bush where the polymer melt is first introduced to the mold. Runner sizing is very important, but often the decision regarding its size is made at the mold design stage, without actually molding the component. Melt flow data obtained in the laboratory, however, can be used to calculate the optimum runner size. A great deal of energy is often wasted in this important process. This is because it is necessary to apply considerable pressure to the polymer melt in order to fill an injection mold quickly. The molds used are often designed without using flow data and as a consequence, runners (the flow channels), are frequently made too large. This means that the high injection molding pressures used are transmitted into the cavity requiring excessive clamping pressures to hold the two mold halves together. If these pressures are reduced, the process will be simpler, cheaper, and more energy efficient to operate. One way of doing this is to size the runners so they are as small as possible, consistent with mold filling and packing. As the runner is often the thickest part of the molding, any reduction in runner cross-section will also reduce the cooling cycle and speed up production. In addition, smaller runners will result in less material wasted or required to be reworked, thus reducing costs.

Runners.

These melt flow channels (the runners) are often of circular cross-section, although trapezoidal sections are common because of their ease of manufacture. The crosssections of these channels are generally constant from the inlet to the exit. Normally polymer melt fills a mold extremely quickly (in less than one second.) and the amount of heat transfer to the mold is relatively small. From a rheological point of view, flow through the runner may be assumed to be the same as that through a circular die and the pressure drop can be calculated. (For a trapezoidal runner, it is assumed that the pressure drop is the same as that for the largest circular runner that fits neatly within the trapezoidal crosssection.)

Injection Pressure.

Most injection molding machines have available approximately 20,000 psi (140 MN²) of injection pressure. Of this about 10,000 psi may be lost in the runner system and still leave enough to pack the injection mold. To size a runner we first select a runner that seems to be representative and calculate the pressure drop through one of that crosssection. If the pressure drop obtained is too large or too small, then the calculation is repeated, using slightly different runner sizes, until the 'correct' amount of pressure drop is obtained. If the inlet pressure is denoted by ΔP and the length of the runner is L then, as the pressure drops linearly from the inlet to the exit, ΔP/L is constant along the whole length of the runner.

Information Required.

For a runner feeding only one cavity the information required is:
• The thermoplastic material to be used for the component.
• The weight of each component.
• The density of the thermoplastic material.
• A flow curve for the material at the melt temperature being used in the process.
In order to calculate the runner size it is necessary to convert the gravimetric values into volumetric values (i.e. g/s into m³/s). To calculate Q, the volumetric flow rate, it is necessary to know the density of the material at the injection molding temperature. This is relatively easy to obtain if an MFR machine, which is capable of doing a method B determination, is available. If not, the advice of the material supplier may be sought. (Generally, the density of a plastic material at molding temperatures may be considered to be roughly 0.75 times that of the room temperature value.) Ideally, the flow properties of the plastic material should be measured over a range of shear rates, using a high shear rate rheometer, to construct a flow curve. It may, however, be possible to obtain such a flow curve from the material manufacturer.

Calculation Of Pressure Drop (ΔP)

The pressure drop in a constant cross-section runner (fed from a reservoir of material in the nozzle or sprue bush) is related to the flow rate, the die dimensions, and the melt viscosity. For a circular cross-section runner it may be calculated by a relatively simple procedure.

Method 1

A. Select a runner diameter. (For example, 6mm/0.25in.)

B. Calculate the runner volume.

C. Calculate the volume of the molding.

D. Calculate the working volume (V) (mold volume + runner volume).

E. Calculate the volumetric flow rate (Q). (Assuming that the mold filling time is 1 second if the value is not known from experience.) If the mold filling time is t seconds then $Q = V/t$. If a 1 second fill time is used, Q will be numerically equal to V but will have different units.

F. Calculate the apparent wall shear rate ($\dot{\gamma}_{w,a}$) from the formula $\dot{\gamma}_{w,a} = 4Q/\pi R^3$.

G. Read off the wall shear stress (τ_w) corresponding to this shear rate value from the flow curve appropriate to the material.

H. Calculate the pressure drop (ΔP – i.e. the pressure developed at the beginning of the runner) from the shear stress by the formula $P = 2L\tau_w / R$ – (where L is the length and R the radius of the runner).

I. If the pressure drop is less than 10,000 psi (70 MNm²) then one should make the runner smaller and if greater than 10,000 psi (70MNm²), then one should make the runner larger. That is, re-size the runner as appropriate. (Since the designer will normally wish to keep the runner as short as possible the main variable here will be the runner diameter.)

Method 2.

This is the same as above except for stage E.

E. Calculate the volumetric flow rate (Q) from the barrel diameter (of the injection molding machine that is to be used) and of the ram speed (linear screw speed) during mold filling. If the cylinder diameter is D and the ram speed is S, then the rate of volumetric displacement by the ram, (which will be the same as that in the mold cavity) – Q will be: $Q = \pi D^2 S/4$.

If it is not known what machine is to be used then it is possible to estimate the required machine size by assuming that the amount of displacement required in the injection barrel (Vi) is about 10% more than V as defined above i.e. $Vi = 1.1V$. This volume will be the ram area times the stroke of the ram. In practice this is usually about 3D so that the volume will be $(\pi D^2/4)3D$ or $3\pi D^3/4$. Thus $1.1V = 3\pi D^3/4$, or $D^3 = 0.467V$ from which the ram diameter D may be immediately found.

Optimizing Runner Sizes

We have shown that the use of flow curves, in association with standard rheological (flow) equations, enables us to determine the pressure loss that occurs when a molten thermoplastic material flows down a channel. We must, however, decide how much pressure can afford to be lost in the runner system that feeds the several cavities. Since an injection-molding machine has normal working pressures of 137-207MN m² (30,000psi), a pressure drop of 69MN m² (10,000psi) may reasonably be lost in the runner system.

Balanced And Unbalanced Runner Systems

The runner layout selected for this calculation is balanced, which means that the distance (or melt flow path) from the sprue entry to the gate entry is identical for each cavity in the mold. (All of the cavities are also assumed to be identical.) Use of a balanced runner system ensures that each cavity is:
1. Filled with molten material at the same time.
2. Subjected to the same amount of injection pressure, so that identical moldings produced in each cavity. The above statements are based upon the assumption that the gate, dimensions, and cooling for each cavity are identical. Unbalanced runner systems require modifications to the individual branch runners and/or to the gates in order to produce identical moldings.

Information Required.

To calculate the optimum size of a balanced runner layout, some details of the mold and part must be known. These are as follows:
1. The number of parts in the mold.
2. The thermoplastic material to be used for the part.
3. The weight of each part.
4. The density of the thermoplastic material. (To calculate the runner size it is necessary to convert the gravimetric values into volumetric values, i.e. g/s into m³/s.). If a flow rate tester is available, which can do Method B testing, the melt density can be easily obtained.

Runner Sections.

The following procedure is used for a runner layout having three sections (i.e. primary, secondary and tertiary runners). The same procedure can be lengthened or shortened to accommodate runner layouts having fewer or more sections. For each runner section, it is necessary to estimate how much plastic material will flow through that particular section or branch. For example, if the primary runner is centrally fed from the sprue, it can be assumed that the flow is split equally between each section. When calculating pressure drops, numerous calculations often need to be done. It is preferable, therefore, to devise a computer program, so that the laborious calculations can be easily handled.

Calculating The Pressure Drop (ΔP).

See the previous section on 'Pressure Drop in a Runner and Runner Sizing', for calculating the volumetric flow rate (Q) Methods 1 and 2.
A. Select a runner diameter.
B. Select the runner configuration (which will establish the lengths of the various runner branches).
C. Name the runner sections.
D. Calculate the runner volume.
E. Calculate the volume of the molded part.
F. Calculate the working volume (V) (feed system volume plus molded part volume).
G. Calculate Q the volumetric flow rate.

Dynisco
From lab to production,
providing a window into the process

H. Calculate the shear rate ($\bullet\gamma$ w,a).
I. Use a flow curve for the material to read off the shear stress corresponding to this shear rate.
J. Calculate the pressure drop (ΔP) in the primary runner.
K. Calculate the volumetric flow rate for each of the secondary runner branches (Q). This will be Q/N where N is the number of secondary runners and Q is the amount of material flowing through the secondary runner.
L. Calculate the shear rate ($\bullet\gamma$ w,a) for the secondary runner.
M. Read off the shear stress from the flow curve.
N. Calculate the pressure drop (ΔP) in the secondary runner.
O. Crosscheck the calculations up to this stage.
P. Calculate the volumetric flow rate (Q) for the tertiary runner. If there are M tertiary runners per secondary runner this will be Q/MN.
Q. Calculate the shear rate ($\bullet\gamma$ w,a) for the tertiary runner.
R. Read off the shear stress from the flow curve.
S. Calculate the pressure drop (ΔP) for the tertiary runner.
T. Add together the pressure drops for each of the runner sections to obtain the total pressure drop for the complete runner system.

Optimization.
The mold designer will naturally try to keep the runner lengths as short as possible. This will largely be determined by the arrangement of mold cavities. A computer program may be written to test a variety of configurations and runner diameters, to determine which will give the best balance of pressure drop and runner scrap. One approach is to make the assumption that a 10,000psi pressure drop in the runners will be acceptable. The best design will be the one that gives this figure with the minimum amount of total runner volume.

Optimizing The Runner Layout For A Multi-Cavity Mold
The techniques, used to calculate optimum runner diameters for a given material/mold combination, can also be used to determine the configuration of the runner layout for a multi-cavity mold. The mold used to produce these components can be laid out in various ways. Each cavity may be fed from a central sprue by means of its own runner or the cavities can be grouped into sets, with each set being fed from a secondary runner. Calculations were performed for the following group sizes using two different thermoplastic material, general purpose (GP) grades of acrylonitrile butadiene styrene (ABS) and polycarbonate (PC).

Size Of Group	Number Of Primary Runners
1	24
2	12

Size Of Group	Number Of Primary Runners
3	8
4	6
6	4
8	3
12	2

Calculations showed that the configuration of 3 cavities by 8 branches (i.e. The 24 cavities are laid out in groups of three. Each group of 3 cavities is fed by its own main runner.) is the most efficient runner design. For this configuration the pressure drop and the material usage, is the least, or most economical, of all the layouts considered.

Different Runner Sizes

The runner diameter can be selected so that (1.) the runner diameter is constant throughout the runner configuration or (2.) the runner diameter is different for each section, or branch, of its configuration. Thus a different diameter is used for each of the primary, secondary, tertiary, and quaternary runners. By reducing the diameter of the secondary, tertiary and quaternary runner sections it is possible to reduce the percentage of material in the runner without significantly changing the pressure drop for the runner configuration.

Specifying the Clamping Force for an Injection Molding Machine.

Since the pressure in a liquid acts in all directions, it follows that when a polymer melt is injected under pressure into a mold, there is a force developed that will tend to open the mold. In trying to assess this mold opening force it is useful to distinguish what it will be at two different instants of time during the molding operation; just before the mold is completely filled, (i.e.; while the melt is still flowing) and at the start of the packing stage, just after the mold has filled.

Mold Opening Forces During Cavity Filling.

During the cavity filling phase there will be pressure losses, due to shear flow, along the entire flow line; from the front of the injection ram, through the barrel, nozzle, sprue, runner, gate and mold cavity. Where there is converging flow, such as in the nozzle areas, extensional flow will contribute to the pressure losses. It is in theory possible to calculate the pressure losses at each stage, but, generally, it is sufficient to divide the total pressure drop (between the injection ram face and the blind end of the mold cavity) - Ptotal into three components:
a) The pressure drop up to the injection nozzle exit (ΔP_n).
b) The pressure drop in runner and sprue (ΔP_r).
c) The pressure drop in the mold cavity (ΔP_m).

The pressure drop up to the injection nozzle exit, ΔP, is obtained by measuring the pressure generated during an air shot at the correct volumetric injection rate (Q). Generally, there is approximately a 10% pressure difference between the injection line pressure and the nozzle pressure.

b) The pressure drop in the runner and sprue (Pr). The pressure drop in the sprue is usually ignored, as the cross-sections here are quite large. Since the sprue is usually at right angles to the runners and mold cavity, its contribution to the mold opening force is negligible.

Pr is calculated by the following procedure:

i. Use Q to determine the shear rate at the runner wall ($4Q/\pi R^3$ where R is the runner radius).
ii. Use the flow curve for the particular polymer at the melt temperature being employed to find the shear stress (τw) corresponding to the shear rate.
iii. Calculate Pr from the equation $Pr = 2L(\tau w)/R$ where L is the length of the runner. (See Pressure drop in a runner and runner sizing.)
d) The pressure drop in the mold cavity (Pm). Assuming that Ptotal is known, then Pm may be obtained by the expression **Pm = Ptotal - ΔP – Pr**
Pm is the pressure drop between the gate and the blind end of the mold just before the mold completely fills. At that moment the pressure at the front end will be zero. As the pressure varies from point to point, it is necessary to find the average pressure over the whole of the projected area of the mold cavity in order to calculate the total mold opening force. Two useful, simple, cases may be identified:
i) A simple rod or bar shaped cavity. Here it may be assumed that the pressure drop changes linearly from one end to the other and the average pressure at any point can be assumed to be Pm/2. The mold opening force due to melt in the mold cavity Fm will be given by
Fm=APm /2
where Am is the projected area of the mold cavity. It is now necessary to determine the mold opening force due to the runner. This will be obtained by multiplying the projected area of the runner by the average pressure in the runner. The projected area will be 2RL and the average pressure will be Pm + Pr /2. Thus the total mold opening force (F) due to cavity and runner will be given by
F = (Am Pm / 2) + 2RL(Pm + Pr / 2).
ii) A center-gated disc mold. For this type of mold, it is estimated that the force is given by
Fm = Pm πR2m [n / (n+2)]
where n is an experimentally determined constant and Rm is the radius of the disc. (Barrie, Plastics and Polymers 37 463, (1970)) Similar arguments may be made for multi-cavity molds involving several runners.

Mold Opening Force After Cessation Of Flow.
During the packing stage there should, in theory, be complete transmission of
pressure between the ram (the screw tip) and the blind end of the cavity – up to the

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moment that the gate freezes. In practice, however, there is some pressure loss. The mold opening force may be calculated from the expression **F = P total A G**

where A is the total projected area of mold cavity plus runners and G is the correction factor. G is usually in the range 0.8-0.9, but may be as low as 0.7 in extended runner systems.

Mold Locking Force.

The above calculations yield estimates of the maximum opening pressure while melt is still flowing and after the mold has filled. The locking force employed, measured in tons, must exceed the higher of these two values. The clamping pressure, measured in tons per square inch, is based upon the projected area of the component and is often of the order of 2.5 tons/ in2. It is, therefore, possible to obtain a good idea of the clamping force required from a relatively simple calculation.

SOLVING PRODUCTION PROBLEMS THROUGH UNDERSTANDING FLOW BEHAVIOR

In this section a number of problems are considered, which can be solved without calculations, but simply with an understanding of flow behavior.

Qualitative Application Of Rheological Concepts

Since most processes for shaping rubbers and plastics involve flow, it might be expected that rheological concepts are widely used in problem solving. In practice, however, quantitative applications are somewhat limited, because of the complex flow behavior of polymer melts, the complex flow patterns involved in shaping operations (e.g. the filling of an injection mold), and the fact that cooling often occurs while the melt is still flowing. Nevertheless, a qualitative understanding of flow behavior can often help to overcome processing problems.

Effect Of Flow Channel Shape

For a flow channel of a fixed cross-sectional area, greater flow rates occur when the channel is circular than when it is in the form of a slit. The narrower the slit (even when the cross-sectional area is the same) the lower the flow rate (under a constant delivery pressure). This difference is relevant if attempts are being made to extrude complex shapes, where the extruded cross-section is made up of components of different thickness and shape. If, for example, the extrudate cross-section is in the form of a dumbbell with two almost spherical parts being bridged by a slit, flow will tend to be faster through of the round sections and slower through the bridging slit. This will lead to stretching, and possibly tearing, of the central section of the extrudate. Even greater problems may occur when the extrudate cross-section is like a half dumbbell, or key cross-section, with one circular section attached to a slit. In this case the extrudate will tend to curl up as it leaves the die. While such designs are best avoided, the problem may be solved by reducing the parallel portion of the die in those regions where the flow is slowest (i.e., in the

slit zones). Throttling the flow in the regions where flow is fastest, by the insertion of a pin or mandrel into the flow channel at the entrance to the die parallel, may also be effective.

Unbalanced Flow In Tubular Extrusion

Tubing is made by extruding polymer through the annular gap between the walls of a circular die and an inserted pin or mandrel. It is sometimes found that the inside wall of the tube has a number of trans-verse ripples. These ripples arise from variations the flow velocity through the die in the radial direction. In a simple rod die the flow rate is highest in the center of the die and zero, or almost zero, at the wall. (With simple liquids, such as water, the velocity profile is parabolic in shape, while with polymer melts it is somewhat flatter). If one were to extrude over a very thin pin (of a thickness approximating to that of a thin piece of wire) the velocity profile would be little affected. (The flow rate a short distance from the thin pin would be much greater than the flow rate the same distance from the die wall.) As the diameter of the pin increases the difference will decrease, but, to a greater or lesser extent, the flowrates will still be higher nearer to the pin than to the die wall. Since the melt is coming out faster on the inside of the tube this will tend to cause buckling and hence rippling on the inside of the tube. One way of reducing this is to fit a restrictor bulge or ring around the pin to slow up the flow in this region.

Uneven Die Swell

As a general rule, die swell increases with shear rate and decreases with the length of the die parallel. If a complex section is being extruded, with different cross-sections, then different shear rates will exist at the walls of the different cross-sections. The shear rates will be highest where the cross-section is the least. It may be possible to compensate for these differences when carrying out die sizing (see the section entitled 'Sizing of Die Components'). However, if it is found that swell at one point is lower than elsewhere, then the die parallel at that cross-section may be reduced to compensate. If this is done, one should machine the back end of the die parallel to ensure that all changes in cross-section are gradual.

Die Swell And Parison Sag During Blow Molding.

In extrusion blow molding it is common to extrude a parison vertically downwards. The tube diameters, while tending to increase due to die swell, will also tend to decrease as the parison sags under its own weight. While the swell effect will be independent of parison length the sag will increase as the tube length increases. It is possible to compensate for this by using a tapered die and pin, which move axially with respect to each other, to open the die gap during the extrusion of the parison. The parison pin movement may be programmed either as the result of trial and error experiments or by use of theoretical data.

Extrudates Exhibiting Melt Fracture Or Sharkskin.

It is often found that surface defects on an extrudate have a regular pattern on the surface. With large diameter extrudates this is likely to be due to sharkskin. With smaller diameter products it is probably to be due to melt fracture. If the defects are transverse to the direction of flow it is probably a form of

sharkskin. If they are helical in form, then it is probably melt fracture. These phenomena and their characteristics are discussed in the section entitled "Elastic Effects in Polymers". If problem is identified as one of melt fracture, then the following possibilities should be considered:

1. Reduce output rate (not usually commercially desirable).
2. Increase the melt temperature (increases cooling times).
3. Increase the taper of the die entry.
4. Taper the 'die parallel' up to 10 degrees.
5. Use a polymer with lower molecular weight.

If the problem is identified as sharkskin, then the following possibilities should be considered:

1. Reduce output rate.
2. Vary the die temperatures.

Either raising or lowering of the die body temperature may help, while extra heating of the die exit may be particularly beneficial.

Short Injection Moldings

Assuming that the injection stroke length has been correctly set (so that the correct mass of material is delivered to the mold), failure to fill the mold cavities arises because the polymer has frozen before filling is complete. Raising the melt temperature will both reduce the melt viscosity and increase the time taken for the melt to freeze. If this is carried to excess, however, the polymer may degrade or the cooling times may be become excessively long. One may also increase the mold temperature, however, this may lead to an unacceptably long cooling time. It also may be possible to use an easier flowing grade, if such a material is available. Failing this, it may be better to re-site the gate to reduce the length of flow path required. Computer programs are available that optimize gate positions and enable visualization of mold filling operations.

SYSTEMS OF UNITS

c.g.s. – Centimeter-gram-second. A system of measurement that uses the centimeter (cm) as the basic unit of length, the second(s) as the basic unit of time, and the gram (g) as the basic unit of mass. The derived unit of force is the dyne (dyn). This system is sometimes known as the CGS or cgs System.

f.p.s. – Foot-pound-second. A system of measurement that uses the foot (ft) as the basic unit of length, the second (s) as the basic unit of time, and the pound as the basic unit of force (force i.e. lbf). The unit of mass is the slug. This system is sometimes known as the FPS gravitational, imperial, or British Engineering system of units. Also known as FPS or fps System.

i.p.s. – Inch-pound-second. A system of measurement that uses the inch (in) as the basic unit of length, the second(s) as the basic unit of time, and the pound force (lbf or psi) as the unit of force. This system is sometimes known as IPS or ips System. m.k.s. - Meter-kilogram-second. A system of measurement that

uses the meter (m) as the basic unit of length, the second(s) as the basic unit of time and the kilogram (kg) as the basic unit of mass. The derived unit of force is the olysu (N). This system is sometimes known as MKS or mks System.

MKSA metric. See S.I.

S.I. – Système International d'Unités. An international system of units now widely used for scientific and technical work. The seven basic units are the meter (length), kilogram (mass), second (time), ampere (electric current), Kelvin degree (temperature), mole (amount of substance) and the candela (luminous intensity). There are supplementary units for plane angle (radian) and solid angle (steradian) and some SI derived units that have special symbols, which include the Newton and Pascal. Certain other non-SI units are permitted, such as cm and ºC. SI units were derived from the MKS system and are sometimes known as MKSA metric

PROPERTY	SYMBOL	SYSTEM OF MEASUREMENT			
		c.g.s.	f.p.s.	i.p.s.	S.I.
Shear Stress	τ	dyn cm²	lbf ft²	lbf in²	Nm²
Shear Rate	$\dot{\gamma}$	s⁻¹	s⁻¹	s⁻¹	s⁻¹
Apparent Viscosity	η	dyn s cm²	lbf s ft²	lbf s in²	Pa-s

VISCOSITY TERMS.
Shear stress is usually designated by the Greek letter τ and has the units of force per unit area. 1 dyn cm² = 0.002 088 lbf ft² = 0.000 014 5 lbf in² or psi. Shear rate is usually designated by the Greek letter gamma with a dot above the letter, that is, gamma dot or . No matter what system of units is used, it is represented by 1/time (in seconds) or reciprocal seconds or s •γ -1.
Apparent viscosity. Dividing τ (the shear stress) by the corresponding (the shear rate) gives the viscosity (η •γ a) at a particular shear rate. This is an apparent viscosity: In practice the word 'apparent' is often omitted. Apparent viscosity has the units of force per unit area times time (in seconds). See Dynamic viscosity. Dynamic viscosity. An alternative name for the coefficient of viscosity of a fluid. The usual symbol is η. In the c.g.s system, the units are dyn s cm² or 'poise' (P). One poise = 1P = 0.1 Pa-s = 0.1 Nsm⁻² = 0.00014 5 lbf s in⁻². One Pascal second = 1 Pa-s = 1 Nsm⁻² = 0.102 kgf s m⁻² = 10 P = 0.00145 lbf s in⁻². Kinematic viscosity. The kinematic viscosity is obtained by dividing the coefficient of viscosity (the viscosity) of a fluid by its density. The units are Stokes = cm2s-1 or ft2s-1.